Debt with Potential Repudiation: A Theoretical and Empirical Analysis

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Debt with Potential Repudiation: Theoretical and Empirical Analysis

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1. INTRODUCTION

The analysis of financial transactions when default and bankruptcy are possible strategies poses difficult theoretical questions. Several researchers have focused on bankruptcy by individual agents in a domestic economy but have not encompassed all issues relevant to borrowing by foreign governments in international private capital markets. Recent large-scale borrowing by governments of poor countries in private markets suggests the need for a further theoretical analysis of situations where repayment is in doubt. This paper provides a theoretical analysis of borrowing by sovereign nations complementing previous research on bankruptcy in domestic financial markets. This theory is then used to specify and estimate an econometric model of borrowing by poor countries in international financial markets.

Our analysis distinguishes between the bankruptcy of an individual economic agent in a national economy and default by a government. In the case of an individual unit, bankruptcy usually reflects negative net worth. Bankruptcy laws provide an institutional framework defining this condition, and creditors are compensated to the extent that assets allow. Domestic bankruptcy laws prohibit an agent from shedding his liabilities while maintaining full control of his assets. The situation is quite different in the case of international lending. Unless the governments of private creditors are willing to coerce debtor governments into repaying loans, there is no explicit mechanism deterring a government from repudiating its external debts. Any net worth criterion is essentially irrelevant. Thus the existence of private loans to foreign governments appears to be a paradox, but can be understood using a model with an endogenous default penalty.

Even without legal or coercive methods of enforcing repayment, private creditors can take a number of retaliatory actions to penalize defaulting debtors. Among the most important of these penalties is exclusion from future borrowing. Of course, the threat of future exclusion will not deter a country from defaulting if it plans to borrow on an uninterrupted basis for a period of time with no further intention of borrowing thereafter. Such a government would be eager to borrow so long as net flows were positive, but should net repayment be required, it would repudiate its debts. Rational lenders, anticipating this behaviour, would never lend to this type of country.

The typical country is, however, unlikely to be a one-time borrower. Countries generally experience periods of high and low income relative to trend, and this variance in their income can play a crucial role in determining the amount they can be lent safely. A country, if it has not defaulted, can borrow in bad periods with the obligation to repay in
good periods. For instance, a country may borrow from international markets during a
crop failure and repay in periods of ample production. The government chooses to repay
because it knows that at some time in the future it may face another shock during which it
will again desire to borrow.

Should the country refuse to repay, we assume that it faces an embargo on future loans
by private lenders and that this embargo is permanent. This assumption is a convenient
theoretical representation of the stylized fact that default makes re-entry into private
capital markets difficult. A country which has defaulted therefore is forever unable to use
international borrowing to smooth its absorption across periods of varying income.

Lenders are assumed to know all relevant characteristics of individual borrowers. One
particular attribute of all borrowers is that they are inherently dishonest in that they will
default if it is to their benefit. Although there may be no exogenous cost of default, lending
is still possible because borrowers optimize over an infinite horizon in which repayment (in
periods of high income) is a condition for borrowing in subsequent periods (of low income).
In one version of our model these periods of high and low income are assumed to alternate
indefinitely. The benefits of default grow with the size of the outstanding debt. The costs,
however, are determined endogenously by the variability and growth rate of the country's
income and several other of its characteristics affecting its future demand for debt. As a
consequence, a maximum safe level of borrowing at which the costs just exceed the
benefits of default is determined. That the penalty of default and implied credit ceiling are
endogenously determined is an important aspect of our paper. 2

In general, a borrower may or may not desire to borrow up to his credit ceiling.
Consequently, the amount which any poor country has actually borrowed is the minimum
of two quantities—the amount it wishes to borrow and the amount it can borrow. Both
amounts are determined by a set of country characteristics elaborated in the theoretical
section of the paper.

In the third section of the paper the factors identified as determinants of the credit
ceiling and the country's desired debt are used to specify an econometric model of
borrowing. This model has an econometric specification formally identical to the maxi-
mum likelihood model discussed by Quandt (1977) for the estimation of markets in
disequilibrium. Consequently, the model can be estimated to allow for the possibility that
any observation may or may not have been generated by credit rationing. Individual
countries can then be classified as credit constrained or unconstrained. The empirical
work indicates that most countries were credit constrained. The variables suggested by the
theoretical section are quite significant in explaining poor country debt.

2. MODELS OF BORROWING WITH DEFAULT
This section develops theoretical models of international borrowing. Section 2.1 provides
a quite general specification of competitive equilibrium in international capital markets. A
characteristic of this equilibrium is that the level of debt may be determined either by the
borrower's demand for credit or by a credit ceiling imposed by the lender. The small
borrower faces a cost of credit curve which is, in general, upward sloping to the point of the
credit ceiling. This conclusion is useful in justifying an econometric model where observed
debt is the minimum of two quantities, desired borrowing and maximum permissible
borrowing.

In Sections 2.2 and 2.3 we explore the role of a number of variables as determinants of
desired borrowing and the credit ceiling. These models are of necessity simpler than the
rather general model of 2.1. Among the factors considered are the level, average growth
rate and percentage variability of the borrower's income, and the level of an exogenous
retaliatory penalty imposed by lender countries on borrowers. Section 2.4 summarizes
those aspects of the theory useful in the econometric specification of Section 3.
2.1. A model of borrowing equilibrium

Assumptions 1 through 3 characterize the borrower:

Assumption 1. Net Output in period $t$, $y_t$, is a random variable with p.d.f. $g_t(y_t)$, for which there exists a maximum output $\bar{y}_t < \infty$ such that $\int_{0}^{\bar{y}_t} g_t(y_t) \, dy_t = 1$.

Assumption 2. Output is not storable so that

$$ c_t = y_t + b_t - p_t \tag{1} $$

where $c_t$ is absorption, $b_t$ is borrowing and $p_t$ is debt-service payments, all in period $t$.

Assumption 3. The borrower's objective function is, with $U$ bounded from above,

$$ E[\sum_{t=0}^{\infty} \beta^t U(c_t - P_t)] \tag{2} $$

where $U' > 0$, $U'' < 0$ and $0 < \beta < 1$. $P_t$ is a penalty imposed for defaulting in addition to the embargo on future borrowing. $P_t$ may arise from a cutoff of aid or retaliatory interference by the creditors or their governments with commodity trade.

Borrowing opportunities are described in Assumptions 4 and 5.

Assumption 4. Debt matures in one period. The repayment function is given by:

$$ d_{t+1} = R(b_t) \tag{3} $$

where $d_{t+1}$ is debt service obligations in period $t+1$. Usually, $d_{t+1} = (1 + r_t)b_t$ where $r_t$ is the interest rate at which $b_t$ is contracted.

Assumption 5. In each period the borrower chooses $b_t \in B_t$ and $p_t$, where $B_t$ is the set of loan amounts available in period $t$. If $p_t < d_t$, so that debt payments fall short of debt obligations, $B_t = \{0\} \forall t \geq t$, i.e. the country is no longer allowed to borrow. Otherwise, if $p_t = d_t$ then $B_t = B(y_t, d_t)$. Obviously, under this assumption, the debtor will choose either $p_t = 0$ or $p_t = d_t$.

Definition 1. The value of the objective function in $t$ given a decision to default in $t$ is

$$ V^D(y_t) = E[\sum_{t=1}^{\infty} \beta^{t-1} U(y_t - P_t)] \tag{4} $$

Definition 2. The value of the objective function in $t$ given a decision not to default in period $t$ is

$$ V^R(y_t, d_t) = \sup_{b_t \in B_t} \{U(y_t + b_t - d_t)$$

$$ + \beta E \max [V^R(y_{t+1}, d_{t+1}), V^D(y_{t+1})]\} \tag{5} $$

where $d_{t+1}$ is given by (3).

We will assume that the value function $V^R$ defined by (5) exists and is unique. Default is optimal in $t$ if and only if

$$ V^D(y_t) > V^R(y_t, d_t). \tag{6} $$

The probability $\lambda$ of default in $t$ as anticipated in $t-1$ is thus given by $\lambda(d_t)$ where

$$ \lambda(d_t) = \Pr [V^D_t > V^R_t]. \tag{7} $$

Theorem 1. The probability of default in period $t$ increases monotonically with debt service obligations $d_t$ in period $t$. 

Assumption 6. Lenders are competitive. They are risk neutral or, alternatively, the risk of an individual default is uncorrelated with market risk, allowing perfect diversification of default risk.

Assumption 7. Lenders can lend to alternative borrowers safely at interest rate $\bar{r}$.

Assumption 8. Lenders know the function $\lambda(d_i)$.

Assumption 9. Total loanable funds are bounded by $W_t < \infty, \forall t < \infty$ so that $\sup B_t \leq W_r$.

Assumption 9 may simply arise from the finiteness of world wealth.

Definition 3. A competitive borrowing equilibrium satisfying Assumptions 1 to 9 is characterized by the function $V^*(y_t, d_t)$ and by the functions $B^*(y_t, d_t)$ and $R^*(b_t)$ defined for $b_t \in B^*$ which maximize $V^*(y_t, d_t)$ subject to the constraint

\[ 1 - \lambda^*[R^*(b_t)]R^*(b_t) = (1 + \bar{r})b_t, \quad \forall b_t \in B^*. \tag{8} \]

Lenders will only make loans which guarantee them an expected rate of return at least as high as the market interest rate. Competition among lenders will ensure that the terms on which these loans are offered maximize borrowers' utility.

Observe that if $\lambda$ is differentiable then $R^*(b_t)$ is defined by the differential equation

\[ R^*(b_t) = (1 + \bar{r})/\{1 - \lambda[R^*(b_t)] - \lambda[R^*(b_t)]R^*(b_t)\}. \tag{9} \]

Since

\[ V^R(y_t, 0) - V^D(y_t) \geq 0, \quad \forall y_t, \]

then

\[ \lambda(0) = 0 \]

so that, from (8),

\[ R^*(0) = 0 \tag{10} \]

constitutes an initial condition for (9). Note that

\[ R^{**}(0) = 1 + \bar{r} \]

so that loans in amounts near zero are charged the safe interest rate.

Some useful properties of this equilibrium are expressed in Theorems 2 and 3 below:

**Theorem 2.** The set of available loan amounts in period $t$ is given by $[0, \bar{b}_t]$ for some $\bar{b}_t < \infty$.

**Theorem 3.** $R^*(b_t)$ is increasing and convex over $[0, \bar{b}_t]$.

These theorems establish that equilibrium borrowing in a credit market determined under Assumptions 1 through 9 will be characterized by an upward-sloping supply curve for credit since $R^*(b_t)$ increases monotonically in this range, and that the total amount of credit available on any terms is bounded.

We define the demand for credit as the level of borrowing $b_t^*$ which the borrower would choose if he could borrow any positive amount. Formally, $b_t^*$ attains the value
function $V^U(y_t, d_t)$ where
\[ V^U = \sup_{b_t \in B^*} \{ U(y_t + b_t - d_t) + \beta E \max [V^R(y_{t+1}, R^*(b_t)), V^D(y_{t+1})] \}. \]

Definition 3 defines $R^*(b_t)$ only for $b_t \in B^*$. Here we define
\[ R^*(b_t) = R^*(\bar{b}_t)b_t/\bar{b}_t, \quad b_t > \bar{b}_t. \]
That is, we assume that the borrower acts as if the rate of interest on $\bar{b}_t$ applies to any hypothetical loan in excess of $\bar{b}_t$. Thus, if the borrower wishes to borrow more than $\bar{b}_t$ at the terms at which $\bar{b}_t$ is offered, $\bar{b}_t^* > \bar{b}_t$ and he is rationed. If $\bar{b}_t^* < \bar{b}_n$, the ceiling is nonbinding.

For the second-order condition for a maximum to be satisfied $U(y_t + b_t - d_t) + \beta E[\max V^R(y_{t+1}, R^*(b_t)), V^D(y_{t+1})]$ must be a concave function of $b_t$. Thus for $b_t < b_t^*$ the objective function is increasing in $b_t$. If the borrower cannot borrow $b_t^*$ he will borrow the largest amount available, $\bar{b}_t$. Actual borrowing $b_t$ is therefore determined by the condition
\[ b_t = \min (b_t^*, \bar{b}_t) \quad (11) \]

To show that rationing may occur, define
\[ \bar{d}_{t+1} = \inf \{ d : 1 - \lambda (d_{t+1}) - \lambda '(d_{t+1})d_{t+1} = 0 \}. \]
If $d_{t+1} > \bar{y}_{t+1} + W_{t+1} < \infty$, repayment is impossible in any state of nature in period $t+1$ and so $\lambda (d_{t+1}) = 1$ for all $d_{t+1} > \bar{y}_{t+1} + W_{t+1}$. Therefore $d_{t+1} < \infty$ and the maximum expected debt service payment in period $t+1$ is $[1 - \lambda (d_{t+1})]d_{t+1} < \infty$. The zero expected profit condition (8) implies
\[ b_t \leq \bar{b}_t = [1 - \lambda (\bar{d}_{t+1})]\bar{d}_{t+1}/(1 + \bar{r}). \]
For values of $b_t > \bar{b}_t$, there exists no repayment obligation which will make loans worthwhile to lenders. No loans in excess of $\bar{b}_t$ will be made. Nevertheless, $b_t^*$ which attains $V^U$ may exceed $\bar{b}_t$ in which case rationing is imposed.

That there exists a maximum amount which can be borrowed follows trivially from the assumption that total loanable resources are finite. This restriction by itself does not, of course, imply rationing. Typically one assumes that prices adjust to allocate finite resources among consumers. In credit markets, however, the borrower will not pay the price if he defaults, and, from Theorem 1, he is more likely to default the more he has borrowed. Thus a non-price allocation mechanism, credit rationing, may arise.

2.2. A deterministic model of borrowing

An explicit solution of the contraction equation (5) requires that $V^R(y_{t+1}, d_{t+1})$ be a simple function of $V^R(y_t, d_t)$ so that $V^R(y_t, d_t)$ can be expressed in terms of $\sup \{ U(y_t + b_t - d_t) \}$. One difficulty in the application of this solution technique is the $E[\max \{ \cdot \}]$ operator in the contraction equation. This operator arises because a decision not to default this period is not a decision never to default, but only a decision to wait one period to reconsider whether default is optimal. Further, in common with most other dynamic programming formulations e.g. (Levhari and Mirman, 1977 and Merton, 1969), only very simple utility functions allow explicit solution of the contraction equation. In this section, we adopt a number of specializations of the model of 2.1 to obtain information on the comparative static response of the model.

We now assume that the borrower’s income alternates between a value which is high relative to trend and a value which is low relative to trend. Borrowing occurs in periods of relatively low income and must be fully repaid in the succeeding period. Failure to repay prevents borrowing in subsequent periods. Note that this analysis is restricted to borrowing only for short-term smoothing purposes; trend income and consumption growth are
the same. The borrower may also lend in international capital markets in high-income periods and liquidate these investments in low periods, a process we refer to as saving. This strategy may be especially relevant after default and can preclude the borrower’s ever being lent any positive amount. As with borrowing, savings are only for short-term smoothing purposes.

Let $y$ denote the initial level of trend income. Let $G = 1 + g$ where $g$ is the growth rate of trend income and let $\sigma$ be the percent by which income deviates from trend. The borrower who has never defaulted borrows a percent $b \geq 0$ of $\sigma$ and saves a percent $s \geq 0$ of $\sigma$ at gross rates of interest $R$ and $R'$ respectively. The borrower therefore consumes $c^t$ in a low period where

$$c^t(s, b, t) = \left[1 - \sigma \left(1 - b - \frac{R's}{G}\right)\right] yG^t, \quad t = 0, 2, 4, \ldots, \quad (12a)$$

and $c^h$ in a high period where

$$c^h(s, b, t) = \left[1 + \sigma \left(1 - \frac{Rb}{G} - s\right)\right] yG^t, \quad t = 1, 3, 5, \ldots, \quad (12b)$$

Values of $b$ and $s$ must be chosen from a set which guarantees that $c^t$ and $c^h$ remain non-negative. A borrower who has defaulted is constrained to $b = 0$. For convenience let $y = 1$ in what follows.

The utility of consumption in each period is given by a constant relative risk aversion utility function:

$$U(X) = \begin{cases} 
X^{1-\gamma} & \gamma > 0, \gamma \neq 1 \\
1-\gamma & \gamma = 1 
\end{cases} \quad (13)$$

The utility of consumption $t$ periods hence is discounted by $\beta^t$ where $0 < \beta < 1$.

We define the two-period discounted utilities $W^h$ and $W^l$ according to whether the perspective is from a high or low period as

$$W^h(s, b) = U[c^h(s, b, t)] + \beta U[c^l(s, b, t + 1)] \quad (14a)$$

$$W^l(s, b) = U[c^l(s, b, t)] + \beta U[c^h(s, b, t + 1)]. \quad (14b)$$

When $c = c^h$ the borrower maximizes $W^h$ with respect to $s$, given $b$. While, when $c = c^l$, he maximizes $W^l$ with respect to $b$, given $s$. This rule determines optimal values of $s$ and $b$ on the assumption of repayment.

An unconstrained borrower will typically choose to smooth absorption by specializing in either lending or borrowing.

**Proposition 1.** If

$$G^\gamma > \beta(R'R)^{1/2} \quad (15)$$

desired borrowing is positive while desired savings is zero. Conversely, if

$$G^\gamma < \beta(R'R)^{1/2} \quad (16)$$

desired borrowing is zero while desired savings is positive. If

$$G^\gamma = \beta(R'R)^{1/2} \quad (17)$$

the borrower is indifferent between smoothing via borrowing or lending.
Proof. First-order conditions for a maximum are

\[
\frac{\partial W_h}{\partial s} = \left[ 1 - \sigma \left( 1 - b - \frac{R's}{G} \right) \right]^{\gamma} - \frac{G^{\gamma}}{\beta R} \left[ 1 + \sigma \left( 1 - \frac{Rb}{G} - s \right) \right]^{\gamma} = 0 \tag{18a}
\]

and

\[
\frac{\partial W_i}{\partial b} = \left[ 1 - \sigma \left( 1 - b - \frac{R's}{G} \right) \right]^{\gamma} - \frac{G^{-\gamma}}{\beta R} \left[ 1 + \sigma \left( 1 - \frac{Rb}{G} - s \right) \right]^{\gamma} = 0. \tag{18b}
\]

If (16) obtains, (18a) will be satisfied when (18b) is negative; a maximum obtains when \( s^* > 0 \) and \( b^* = 0 \). Conversely, if (15) obtains, (18b) will be satisfied when (18a) is negative; a maximum obtains when \( b^* > 0 \) and \( s^* = 0 \).

An equilibrium with positive borrowing will be observed only when the growth rate is large relative to the discount factor and the geometric average of the borrowing and lending rates. If the growth rate is positive, borrowing is more likely to be observed when the elasticity of marginal utility is large.

If condition (15) obtains then the desired amount of borrowing relative to income is:

\[
\sigma b^* = \frac{\sigma [1 + (\beta R)^{-1/\gamma} G] + [(\beta R)^{-1/\gamma} G - 1]}{[1 + (\beta R)^{-1/\gamma} R]}
\]

implying

**Proposition 2.** Desired borrowing depends positively on the growth rate \( G \) and the standard deviation of income \( \sigma \).

Having determined the desired level of borrowing given a decision not to default in the repayment period, we now define a sustainable debt level as one which the borrower will repay in periods of high income if he may borrow that amount in periods of low income. Discounted utility given a decision to repay \( Rb \) when \( c = c^h \) to maintain the ability to borrow \( b \) in periods in which \( c = c^i \) is given by

\[
V^R(b) = \sum_{t=0}^{\infty} \beta^{2t} \max_{s \geq 0} \left[ W^h_t(s, b) \right]. \tag{20a}
\]

Since from (18a) \( s^* \) is independent of \( t \), the recursive nature of dynamic programming and (20a) imply

\[
V^R(b) = \left[ \max_{s \geq 0} W^h(s, b) \right] \phi^{-1}
\]

where

\[
\phi = 1 - (\beta G^{1-\gamma})^2
\]

which we assume positive to ensure boundedness.

In order to characterize the set of sustainable debt levels it is useful to demonstrate the following two properties of \( V^R(b) \).

**Lemma 1.** At all values of \( b > 0 \) such that \( V^R(b) = V^R(0) \) or \( V^R(b) = 0 \), the optimal level of saving is zero.

Proof. Consider the lowest value of \( b > 0 \) such that \( V^R(b) = V^R(0) \), denoted by \( \bar{b} \). Since \( V^R(b) \) is a continuous function it must attain either a local maximum or a local minimum at some point \( \bar{b} \), where \( 0 < \bar{b} < b \). We show first that optimal savings at \( \bar{b} \) is zero. We then show that optimal savings \( s^* \) is a monotonically decreasing function of \( b \).
The first part of this result obtains via an argument analogous to that used to establish Proposition 1. If $G > (R'R)^{1/2}$ then $\bar{b}^* > 0$ while $s^* = 0$ and conversely. Optimal savings is zero at any extremum. The second part is established by observing that optimal savings as a function of $b$ is given by

$$s^*(b) = \max \left\{ \frac{[1 - (\beta R')^{-1/2}G + \sigma[1 - (Rb/G) + (\beta R')^{-1/2}G(1-b)]}{\sigma[1 + R'(\beta R')^{-1/2}]}, 0 \right\}. \quad (21)$$

Thus if $s^*(\bar{b}^*) = 0$, $s^*(b) = 0$ for all $b \geq \bar{b}^*$, including $\bar{b}$. ||

**Lemma 2.** There exists at most one $\bar{b} > 0$ such that $V^R(b) = V^R(0)$.

**Proof.** Assume the contrary. Then since $V^R(b)$ is continuous, it must attain both at least one maximum and one minimum at strictly positive values of $b$. However, by Lemma 1 $s^*(b) = 0$ for all values of $b$ above the first local extremum. Hence $V^R(b) = W^h(0, b)\phi^{-1}, \forall b \geq \bar{b}^*$. Since $W^h(0, b)\phi^{-1}$ is strictly concave function of $b$ it can attain no minima and at most one maximum. Hence $V^R(b) = V^R(0)$ for at most one $\bar{b} > 0$. ||

We can now demonstrate:

**Proposition 3.** The set of sustainable debt levels may be characterized by a finite interval $[0, \bar{b}]$.

**Proof.** By Lemma 2 $V^R(b) = V^R(0)$ at most once. If $V^{R'}(0) > 0$ then

$$\bar{b} = \min (\bar{\bar{b}}, \bar{b})$$

where

$$\bar{\bar{b}} = G(1 + \sigma)/R\sigma,$$

the highest value $b$ can attain which permits $c^h \geq 0$ and $s \geq 0$, while

$$b = b: V^R(b) = V^R(0).$$

A sustainable debt level $b$ must satisfy

(i) $V^R(b) - V^R(0) \geq 0$, (ii) $c^h \geq 0$, and (iii) $s \geq 0$.

If $b > \bar{b}$ then (i) is violated while if $b > \bar{b}$ or (iii) must be violated. If $b < \bar{b}$ then (i), (ii) and (iii) are satisfied. If $V^{R'}(0) < 0$ then $V^R(b) < V^R(0) \forall b > 0$. No borrowing at a positive level is sustainable and $\bar{b} = 0$. ||

These results are illustrated in Figure 1 where $V^R(b)$, discounted utility from the perspective of a high-income period, $V^l(b)$, discounted utility from the perspective of a low-income period, and $s^*(b)$, optimal savings, are graphed as functions of $b$. $V^R(b)$ achieves a maximum at $\bar{b}^*$, $V^l(b)$ at $b^*$, and $V^R(\bar{b}) = V^R(0)$. $s^*(b)$ is downward sloping and negative at $\bar{b}^*$. In general $b^* > \bar{b}^*$ but $\bar{b}$ may either exceed or be exceeded by $b^*$, a result we have verified by numerical simulation.

To avoid default, lenders will impose a maximum of $\bar{b}G'\sigma$ on loans. Since utility is increasing in $b$ for $b < b^*$, in periods of low income, actual borrowing is given by $\min (G'\bar{b}^*\sigma, G'b^*\sigma)$. If $\bar{b} < b^*$ the borrower is rationed and borrows the maximum sustainable amount, $G'\bar{b}^*\sigma$.

The following propositions characterize the effects of changes in $\sigma$ and $G$ on the credit ceiling.

**Proposition 4.** The credit ceiling is an increasing function of income variance $\sigma$. 
Proof. (i) First consider the case in which \( \bar{b} = \tilde{b} \). Define

\[
\Delta \equiv V^R(\bar{b}) - V^R(0).
\]  

Then

\[
\frac{d\bar{b}}{d\sigma} = \frac{d\tilde{b}}{d\sigma} \sigma + \tilde{b} = -\left( \frac{d\Delta}{d\sigma} / \frac{d\Delta}{db} \right) \sigma + \bar{b}
\]  

(23)

which, upon substituting \( dV^R(0)/ds = 0 \), becomes

\[
\frac{d\tilde{b}}{d\sigma} = \sigma \left( \left\{ \left[ 1 + \sigma \left( 1 - \frac{R\bar{b}}{G} \right) \right]^{-\gamma} - \left[ 1 + \sigma (1 - s) \right]^{-\gamma} \right\} 
- \beta G^{1-\gamma}\left\{ \left[ 1 - \sigma (1 - \tilde{b}) \right]^{-\gamma} - \left[ 1 - \sigma \left( 1 - \frac{R's}{G} \right) \right]^{-\gamma} \right\} \right) / \left( -\frac{d\Delta}{d\tilde{b}} \phi \right)
\]  

(24)

where \( s \) is given by (21) evaluated at \( b = 0 \).

Since \( V^R(b) \) is decreasing in \( b \) at \( \tilde{b} \), the denominator of (24) is positive. A sufficient condition for the numerator to be positive is that

\[
\frac{R\bar{b}}{G} \geq s \quad \text{and} \quad \tilde{b} \geq \frac{R's}{G}.
\]  

(25a)
The condition that $V^R(\tilde{b}) = V^R(0)$ implies

$$\left\{ \left[ 1 + \sigma \left( 1 - \frac{R\tilde{b}}{G} \right) \right]^{1-\gamma} - \left[ 1 + \sigma (1-s) \right]^{1-\gamma} \right\} + \beta G^{1-\gamma} \left\{ \left[ 1 - \sigma (1 - \tilde{b}) \right]^{1-\gamma} - \left[ 1 - \sigma \left( 1 - \frac{R's}{G} \right) \right]^{1-\gamma} \right\} = 0 \quad (26)$$

which is true only if condition (25a) obtains or if

$$\frac{R\tilde{b}}{G} \leq s \quad \text{and} \quad \tilde{b} \leq \frac{R's}{G}. \quad (25b)$$

A strictly positive $\tilde{b}$ requires, however, that $V^R(b)$ be increasing at $b = 0$ and decreasing at $b = \tilde{b}$, i.e. that

$$- \left[ 1 + \sigma \left( 1 - \frac{R\tilde{b}}{G} \right) \right]^{1-\gamma} \frac{R}{G} + \beta G^{1-\gamma} \left[ 1 - \sigma (1 - \tilde{b}) \right]^{-\gamma} \leq 0 \quad (27)$$

$$- \left[ 1 + \sigma (1-s) \right]^{1-\gamma} \frac{R}{G} + \beta G^{1-\gamma} \left[ 1 - \sigma \left( 1 - \frac{R's}{G} \right) \right]^{-\gamma} \geq 0$$

which cannot hold if condition (25b) obtains. Then condition (25a) must obtain, implying that (24) is positive.

(ii) If $b = \tilde{b}$

$$\sigma \frac{db}{d\sigma} = \frac{G}{R} > 0.$$

If $\tilde{b} = \bar{b}$, a borrower subject to greater income variability suffers more from a credit embargo. The penalty of default is therefore greater and lenders can safely lend him larger amounts knowing his increased reluctance to suffer an embargo. If, instead, $\tilde{b} = \bar{b}$, the borrower is constrained in his borrowing by the total resources he has available in periods of high income. As $\sigma$ rises resources available for repayment in high income periods also rise.

**Proposition 5.** An increase in the growth rate $G$ has an ambiguous effect on the credit ceiling.

First consider the case in which $\tilde{b} = \tilde{b}$. In this case the analytic expression for $d\sigma/dG$ is extremely complicated and may be either negative or positive, a fact verified by numerical examples. A negative effect of $G$ on $\bar{b}\sigma$ is more likely the higher $\gamma$, the elasticity of the marginal utility of income; a high growth rate increases the extent to which future penalties are discounted. Furthermore, a negative $d\sigma/dG$ is more likely when relative risk aversion is falling or when income variability falls in proportion to income as income rises. The former conjecture is supported by a simulation analysis using the utility function $U(X) = X^{1-\gamma}/(1-\gamma) + \alpha X$ and observing the effect of increases in $\alpha$ on $d\sigma/dG$. Secondly, if $\tilde{b} = \bar{b}$ then $d\sigma/dG = (1 + \sigma)/R > 0$.

The comparative static effects of changes in other variables on the credit ceiling can be obtained. It is easy to show that $d\sigma/dR \leq 0$; a higher interest rate on savings, making smoothing via borrowing relatively less attractive, reduces the amount which may be safely lent. If a penalty $P$ is imposed in the event of default, such as a cut-off of trade or aid, $d\sigma/dP \equiv 0$ since increases in $P$ decrease $V^D$.

2.3. A stochastic model of borrowing with default

The equilibrium of 2.2 is characterized by the absence of default. Unless lenders err by making loans in excess of $\tilde{b}$, debts will always be repaid in full. This conclusion is not
surprising in a deterministic model in which agents’ expectations are rational. Of course, misperception by lenders of borrowers’ characteristics could lead to default and may, in reality, pose a threat to the stability of the market. This section introduces an alternative reason for default by introducing uncertainty about the borrower’s situation in the period in which loans come due. The introduction of uncertainty complicates the model somewhat and we make offsetting simplifications by assuming no growth \( (G = 1) \) and no saving \( (R' = 0) \). Introducing stochastic income makes it impossible to derive exact analytic expressions from the constant relative risk aversion utility function. For this reason we pursue the analysis of this section by using Taylor-series approximations of a general utility function. The introduction of uncertainty qualifies some results in 2.2. In particular \( d\sigma / d\sigma \) may be negative; that is, an increase in the variability of income may reduce the debt ceiling.

We now assume that the borrower’s income in any future period takes on values \( y^l = 1 - \sigma \) and \( y^h = 1 + \sigma \) with equal probability regardless of income in the previous period. Borrowing opportunities are described in Assumptions 10 and 11.

**Assumption 10.** Borrowing may occur in period \( t \) if and only if \( y_t = y^l, b_{t-1} = 0 \) and the borrower has not previously defaulted.

**Assumption 11.** Loans are due in one period regardless of income in that period.

These assumptions along with the assumptions of competition and risk neutrality among lenders, imply that borrowers may borrow at the safe interest rate \( \bar{r} \) only if they are willing to repay in the following period regardless of the actual value of income in that period. The maximum amount that will be lent at rate \( r \) is thus determined by the largest amount borrowers will be willing to pay in any foreseeable situation in the second period.

Given that they will repay, unconstrained borrowers will select \( b \) to maximize

\[
U[1 - \sigma(1 - b)] + \beta \{U[1 + \sigma(1 - Rb)] + U[1 - \sigma(1 + Rb)]\}/2. \tag{28}
\]

Approximating this expression by a second-order Taylor series expression about \( c = 1 \) yields, as an expression for optimal borrowing,

\[
\sigma b^* = \frac{(1 - \beta R)/A + \sigma}{(1 + \beta R^2)} \tag{29}
\]

where \( A = -U''/U' \), the degree of absolute risk aversion.

The value of continued access to capital markets for a safe loan in amount \( b \) is given by the difference between expected discounted utility with borrowing in that amount under the conditions Assumptions 10 and 11 and expected discounted utility under financial autarchy. Clearly, a given debt-service payment reduces utility more when \( y = y^l \) than when \( y = y^h \). Default is more tempting in hard times. We now define \( V^R(b) \) as the expected discounted utility of borrowing \( b \) whenever possible (i.e. whenever \( y = y^l \) and previous debt is zero) and always repaying. The expected discounted utility of meeting a debt service obligation \( Rb \) in a period in which \( y = y^l \) is thus

\[
U[1 - \sigma(1 + Rb)] + \beta V^R(b) \tag{30}
\]

where, via a transformation based on the recursiveness of the dynamic program,

\[
V^R(b) = \frac{\{U[1 - \sigma(1 - b)] + U(1 + \sigma)\}/2 + \beta \{U[(1 - \sigma(1 + Rb)] + U[1 + \sigma(1 - Rb)]\}/4}{1 - \beta/2 - \beta^2/2}. \tag{31}
\]

The expected discounted utility under autarchy is

\[
V^D = \frac{[U(1 + \sigma) + U(1 - \sigma)]/2}{(1 - \beta)}. \tag{32}
\]
Payment will be optimal if and only if

\[ U[1 - \sigma(1 + Rb)] + \beta V^R(b) - U[1 - \sigma] - \beta V^D \geq 0. \]  

(33)

Approximating this expression by a second-order Taylor series about \( c = 1 \) reveals that payment will be optimal whenever \( 0 \leq b \leq \bar{b} \) where

\[ \sigma \bar{b} = \frac{2\beta(1 + R) - 4R}{A + 2(\beta - 2R\alpha)\sigma}{[\beta(1 + \beta R^2) + 2R^2 \sigma]} \]  

(34)

and

\[ a = 1 - \beta/2 - \beta^2/2. \]

In contrast with the deterministic case, the sign of the effect of changes in \( \sigma \) and \( A \) on \( \sigma \bar{b} \) is ambiguous. For values of \( \beta \) near 1, however, a positive relationship is assured in each case while only if \( \beta \) is close to 0 will a negative one arise. Large values of \( \sigma \) and \( A \) imply that, while the benefits of continued access to capital markets are greater, the current disutility of making a given debt-service payment if \( y = y^h \) is also greater. When the future is heavily discounted the effect of increased variability and risk aversion on the capacity to bear riskless debt is therefore negative.

At an interest rate larger than \( R \) creditors will be willing to extend credit even if there is a positive probability of default. If debtors find it worthwhile to repay a debt if and only if \( y = y^h \), a loan will be repaid with probability one-half. At interest rates greater than or equal to \( 2R \) lenders will find such loans worthwhile.

If the borrower accepts a loan in amount \( b \geq \bar{b} \) in a period in which \( y = y^t \) he will repay in the subsequent period if and only if \( y = y^h \). If \( y = y^t \) for two consecutive periods, then, the borrower will default, excluding himself from the credit market thereafter. Alternatively, if \( y = y^h \) in the repayment period, the borrower must find repayment worthwhile.

At interest rate \( R^* = 2R \) the constrained borrower intending to repay only in the event \( y = y^h \) will choose \( b \) to maximize

\[ U[1 - \sigma(1 - b)] + \beta U[1 + \sigma(1 - R^*b)]/2. \]  

(35)

Optimal borrowing may be approximated by

\[ \sigma b^* = \frac{(1 - \beta R)/A + (1 + \beta R)\sigma}{1 + 2\beta R^2}. \]  

(36)

The borrower must, find it worthwhile to repay \( Rb \) when \( y = y^h \), or borrowing is impossible. Expected discounted utility given repayment is

\[ U[1 + \sigma(1 - R^*b)] + \beta V^R(b) \]  

(37)

where we now define \( V^R(b) \) as the expected discounted utility of borrowing \( b \) whenever possible (i.e. whenever \( y = y^t \) and previous debt is zero) and repaying whenever \( y = y^h \) but not when \( y = y^t \).

From a transformation similar to that used to obtain (31) from (30),

\[ V^R(b) = \{2(U[1 - \sigma(1 - b)] + U[1 + \sigma]) + \beta(U[1 - \sigma] + U[1 + \sigma(1 - R^*b)]) + \beta V^D \}/4e \]  

(38)

\[ \epsilon = 1 - \beta/2 - \beta^2/4 \]  

(39)

and \( V^D \) is the expected discounted utility under autarchy.

Repayment in the case \( y = y^h \) requires

\[ U[1 + \sigma(1 - R^*b)] + \beta V^R(b) - U[1 + \sigma] - \beta V^D \geq 0. \]  

(40)
Approximation by a second-order Taylor series expansion around \( c = 1 \) implies that (40) is satisfied if and only if \( 0 < b \leq \bar{b} \) where

\[
\sigma \bar{b} = \frac{[f - R(2 + \beta f)]/A + [R(2 + \beta f) + f]\sigma}{[R^2(4 + 2\beta f) + f]/2}
\]  

(41)

and \( f = \beta/2e \). Note that \( \sigma \bar{b} \) depends positively on \( \sigma \).

Several types of equilibria are possible in the stochastic model. If, on the one hand, \( b \leq \bar{b} \), which is possible since risky borrowing requires a higher interest rate, only safe loans will be made. These may or may not be subject to rationing. If, on the other hand, \( \bar{b} > \bar{b} \) lending may be either safe or risky and may or may not be constrained by \( \bar{b} \) or \( \bar{b} \) respectively. The specific outcome will depend on the borrower's optimal position given the two-tier credit supply schedule.

2.4. Some implications for econometric specification

In this sub-section we bring together various aspects of the theoretical models relevant to the econometric specification of Section 3:

1. The models suggest that the debt which any country is observed to contract is the minimum of two quantities—the amount it wishes to borrow and its credit ceiling.

2. Increases in the percent variability of income increase the amount a country wishes to borrow. Our presumption is that increases in variability also increase the credit ceiling as illustrated in 2.2, but this result need not always obtain, as found in 2.3.

3. Increases in the growth rate of income increase the amount a country wishes to borrow. Increases in the growth rate may either increase or decrease the credit ceiling. A negative effect of growth on the ceiling is more likely the more risk averse is the country, the more rapidly its risk aversion falls with increases in income and, possibly, if the percent variability of income falls as income rises.

4. If the utility function exhibits constant relative risk aversion then the income elasticities of both the ceiling and desired debt are one.

5. Increases in an exogenous penalty increase the amount a country can borrow.

Although the time series experience is too short for systematic inferences, it seems that much borrowing by poor country governments during the 1970's has been motivated by short-term adjustments associated with oil price increases and OECD recessions. This view of events is broadly consistent with our theoretical framework and its emphasis on smoothing. Indeed, it would be surprising if commercial banks were engaged in long-term lending in international markets, a role they have rarely assumed in a domestic context. On the other hand, our theoretical framework incorporates a secular growth in income and higher growth, for the usual Fisherian reasons, raises desired debt.

3. ECONOMETRIC ANALYSIS

The possibility of credit rationing raised by the theoretical models has important implications for an econometric specification of the relation between the debt a country has contracted and the country's characteristics. The debt of countries in the sample can be either credit ceiling (\( \bar{b} \)) or demand (\( b^* \)) determined. It would be inappropriate to estimate a single relation between debt and a set of independent variables if the sample contains countries whose debt levels were determined by different regimes. The model is of the form

\[
\begin{align*}
  b^* &= f(x) + u \\
  \bar{b} &= g(x) + v \\
  b &= \min (b^*, \bar{b})
\end{align*}
\]  

(42)
where \( b \) is observed debt, \( x \) is a vector of country characteristics, and \( u \) and \( v \) are random errors. The essential aspect of this model is that only \( b \) is observed; it is not known whether any particular \( b \) is a \( b^* \) or a \( \hat{b} \). The joint estimation of the parameters of \( f(\cdot) \) and \( g(\cdot) \) is possible by maximum likelihood methods, due to Maddala and Nelson (1974), requiring nonlinear optimization. Appendix B contains a statement of this likelihood problem.

The data base is a cross-section of countries in two years, 1970 and 1974. All countries in the sample and the years for which each could be included are listed in Appendix D. The size of the sample was solely determined by the availability of all relevant data for any poor country in each of the two years.

Both total supply of and demand for debt of poor country governments in private capital markets are hypothesized as functions of: the percent variability of exports (\( \sigma_x \)), the ratio of imports to GNP(\( M/Y \)), the growth rate of GNP(\( GY \)), total real GNP(\( Y \)), total population (\( POP \)), the real level of debt to public institutions (\( PUBDT \)), and a dummy (\( D \)) equal to zero in 1970 and one in 1974. The first three variables, being percentages or fractions are entered linearly while the next three variables, as well as actual debt (\( PRIDT \)), are entered as natural logarithms.

Parameter estimates are presented in Table I. Both the credit ceiling and desired debt

<table>
<thead>
<tr>
<th>Variable</th>
<th>( b^* ) (supply) Regime</th>
<th>( b^* ) (demand) Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>MLE/SE</td>
</tr>
<tr>
<td>PRIDT</td>
<td>13.5</td>
<td>2.30</td>
</tr>
<tr>
<td>( \sigma_x )</td>
<td>4.88</td>
<td>2.52</td>
</tr>
<tr>
<td>( M/Y )</td>
<td>4.35</td>
<td>2.88</td>
</tr>
<tr>
<td>( Y )</td>
<td>1.18</td>
<td>6.46</td>
</tr>
<tr>
<td>( POP )</td>
<td>-0.64</td>
<td>3.54</td>
</tr>
<tr>
<td>( GY )</td>
<td>-0.12</td>
<td>2.60</td>
</tr>
<tr>
<td>( PUBDT )</td>
<td>0.63</td>
<td>3.14</td>
</tr>
<tr>
<td>( D )</td>
<td>-0.57</td>
<td>2.14</td>
</tr>
</tbody>
</table>

MLE/SE: Maximum likelihood estimate divided by its standard error.
Number of observations = 81.
Value of the log of the likelihood function = -95.2.
Estimate of error variance = 0.705.

are functions of the same set of variables. Consequently, prior restrictions on the parameters are required to make a correspondence between the two theoretical regimes and the two sets of parameters estimated by maximizing the likelihood of Appendix B. As suggested in 2.2 and 2.4, a higher growth rate of income raises desired debt for the usual Fisherian reasons, i.e. some of the future higher income is desired now. On the other hand, higher growth may or may not raise the credit ceiling. A borrower with rapidly growing income may have less to fear from the future effects of a credit embargo, lowering \( b \). Because one set of parameters has a positive effect of growth on debt and the other a negative effect, we identify the former as the demand (\( b^* \)) equation and the latter as the supply constraint (\( \hat{b} \)) equation. Other aspects of the estimation results, discussed below, reinforce this correspondence.

The costs of default arise from retaliation by the international community. If this retaliation takes the form of an embargo on future lending it is likely to be more of a deterrent the higher is export variability.\(^5\) The amount a country wishes to borrow is also
likely to increase with variability. Turning to Table I, the estimated coefficients imply that both \( \hat{b} \) and \( b^* \) rise with increases in \( \sigma_r \). This result supports our basic theoretical framework which provides an interpretation of the positive relation between \( \hat{b} \) and \( \sigma_r \). Frank and Cline (1971) and Feder and Just (1977a), studying defaults on debt owed to governments and international bodies by poor countries, have reported that the probability of default is lower when export revenues are more variable. These researchers regard this result as anomalous, and reject it on a priori grounds. In contrast, we view their results as corroborating our empirical and theoretical findings.

Creditors and their governments may also interfere with foreign trade, especially through the disruption of arrangements for financing trade. The more important are imports \((M/Y)\) the more harmful is trade-related retaliation and the higher is the debt level which can be sustained. We therefore interpret \( M/Y \) as a possible proxy for \( P \), the penalty considered in Section 2. Consistent with this view, the effect of the variable \( M/Y \) is strongly positive in the \( \hat{b} \) regime. We entered \( M/Y \) in the \( b^* \) equation to reflect transactions effects, since some debt to private lenders consists of suppliers credits and other borrowing used to finance trade. This variable has a negative effect on \( b^* \), an anomalous result.\(^6\)

Theory suggests that the levels of both \( \hat{b} \) and \( b^* \) will be positively related to income, possibly with a unitary elasticity if the utility function exhibits constant relative risk aversion. The estimated elasticity of \( \hat{b} \) with respect to \( Y \) is insignificantly different from one. The level of \( b^* \), however, is not related to \( Y \), a surprising result.

Since \( PRIDT, Y, PUBDT \) and \( POP \) are measured in logarithms, the inclusion of \( POP \) allows for the possibility that \( PRIDT, Y \) and \( PUBDT \) should be specified in per capita terms. For the \( \hat{b} \) equation, the sum of the coefficients on \( Y, PUBDT \) and \( POP \) approximates one. Therefore this relation between \( PRIDT \) and \( Y \) and \( PUBDT \) could actually be specified in per capita terms. This result does not hold for the \( b^* \) equation.

We view debt to rich country governments and international agencies as predetermined by political and other considerations not part of the economic decision-making process of a poor country and therefore include \( PUBDT \) as an exogenous variable in both equations. To some degree, \( PUBDT \) substitutes for debt to private markets and lowers \( b^* \). \( PUBDT \) might also lower the safe credit ceiling on private debt. Indeed, if the role and terms of \( PUBDT \) were identical to those of \( PRIDT \), the theoretical models could be viewed as determining total debt desired and allowable. An increase in \( PUBDT \) would then imply a corresponding decrease in \( PRIDT \) whether demand or supply determined. Certainly, the negative sign of \( PUBDT \) in the \( b^* \) equation, although insignificant, does suggest that the two types of debt are substitutes from the borrower's view point. In the \( \hat{b} \) equation, however, \( PUBDT \) has a significantly positive effect. Private lenders may regard a high value of \( PUBDT \) as indicating that public lenders view the country to be generally stable. In addition, a high value of \( PUBDT \) may imply a general commitment by these public institutions to the country. In this case, private lenders may expect public institutions to act as lenders of last resort if insolvency arises. Since the positive effect of \( PUBDT \) is understandable in the context of the \( \hat{b} \) regime, this result is additional evidence that the sets of coefficients correspond to the indicated theoretical regimes.

The variable \( D \) is included to allow for a year-specific component in the error term. It allows us to test the general presumption that, other things equal, the post-OPEC period was characterized by significantly higher borrowing by the average country. The time dummies indicate that credit ceilings were lower in 1974 than in 1970, while the demand for debt was insignificantly higher in 1974 than in 1970, all other factors held constant.\(^7\)

In addition to including the time dummy, we felt that special caution should be taken in merging the 1970 and 1974 cross-sections since the two periods were potentially so different. Consequently, two additional sets of equations were estimated, one on each annual cross-section. A likelihood ratio test analogous to the Chow test in ordinary least squares revealed that the two equations differ only in their intercepts.\(^8\) The null hypothesis
of homogeneity could not be rejected at any conventional level of significance, justifying the pooling of the 1970 and 1974 samples. Given the possible structural instability of the period, the outcome of this test is of considerable interest in itself and supportive of the chosen specification.

One question of special interest is the probability that any particular country in the sample has debt which is supply rather than demand determined. The estimates of these probabilities, computed on the basis of the values of the explanatory variables and the realized value of debt according to the formula in Appendix B, appear in Appendix D. Sixty-five of the eighty-one observations are more likely to be supply constrained; the remaining hold debt which is demand determined. It is reassuring that most observations are assigned to the $b$ regime because credit ceilings (country limits in market terminology) are believed to have been prevalent in these markets during the early nineteen-seventies. The relatively few observations assigned to the $b^*$ regime undoubtedly contributes to the weak significance of several explanatory variables in this regime.\(^9\)

4. CONCLUSIONS

This paper analyzes the recent phenomenon of poor country borrowing in international capital markets. A crucial characteristic of this borrowing is the absence of explicit penalties for non-payment. Instead, borrowers who repudiate their debt face future exclusion from capital markets. Working with the assumption that this exclusion is permanent, we have been able to show that lenders will establish a credit ceiling above which they will be unwilling to increase loans. The amount of this ceiling is determined by lenders' perception of borrowers' disutility of exclusion. If the ceiling is below the amount a borrower wishes to obtain then the borrower is rationed. Our theoretical model relates both the credit ceiling and the demand for credit to a set of observable borrower characteristics. One important elaboration of the theory would be to make the period of exclusion endogenous and to examine the determinants of its length.

Using this theoretical framework as a guide, we specify and estimate the relation between loan demand and supply and a set of country characteristics. The empirical results coincide quite closely with the structure of our theory. Most interestingly, we find that variability of export revenue increases both the demand for and supply of debt. An important extension of our empirical work would be to incorporate dynamic adjustment into the behaviour of participants in the market using time series data on individual borrowers. Only recently has a sufficiently long record for these markets emerged to make this line of research possible.

APPENDIX A

Proof of Theorem 1. Consider any pair of debt service obligations $d^1_t$ and $d^2_t$ such that $d^2_t \geq d^1_t$. Then, where $b^1_t$ and $b^2_t$ attain $V^R(y_t, d^1_t)$ and $V^R(y_t, d^2_t)$ respectively,

\[
V^R(y_t, d^1_t) = U(y_t + b^1_t - d^1_t) + E\beta V(y_{t+1}, R(b^1_t)) \\
\geq U(y_t + b^2_t - d^1_t) + E\beta V(y_{t+1}, R(b^2_t)) \\
\geq U(y_t + b^2_t - d^2_t) + E\beta V(y_{t+1}, R(b^2_t)) \\
= V^R(y_t, d^2_t)
\]

Thus $V^R$ decreases monotonically in $d$, while $V^D$ is independent of $d$. Therefore, from (7), $\lambda$ increases monotonically with $d$.

Proof of Theorem 2. That $\bar{d}_i < \infty$ follows directly from Assumption 6 since $\bar{d}_i < W_i < \infty$. To show that $\forall x$ s.t. $x < \bar{d}_i$, $x \in B^*$ note that $b_i$ must satisfy (8). Lenders would be
willing to offer a loan in any amount $b_i < \hat{b}_i$ in exchange for a debt service obligation of $R^*(b_i)$ in $t+1$ since such a loan would yield excess expected profits. ||

Proof of Theorem 3. Consider any pair of loan amounts $b_1^2, b_2^2 \in B^*$ s.t. $b_2^2 \geq b_1^2$. From Definition 3

$$[1 - \lambda^*(R^*(b_1^2))]R^*(b_2^2) = (1 + \bar{r})b_1^2 \geq (1 + \bar{r})b_1^1 = [1 - \lambda^*[R^*(b_1^1)]]R^*(b_1^1).$$

(A.2)

$R^*(b_1^2)$ constitutes a repayment for $b_1^1$ which provides excess expected profits so that $R^*(b_1^2) \geq R^*(b_1^1)$.

Thus $R^*(b_i)$ is increasing.

From condition (8)

$$R^*(b_1^2)/b_1^2 = (1 + \bar{r})/[1 - \lambda^*[R^*(b_1^2)]].$$

(A.3)

Note the value $x$ where

$$x = (1 + \bar{r})b_1^1/[1 - \lambda^*[R^*(b_1^2)]].$$

(A.4)

satisfies

$$[1 - \lambda^*(x)]x \geq (1 + \bar{r})b_1^1$$

(A.5)

since

$$R^*(b_1^2) \geq (1 + \bar{r})b_1^2/[1 - \lambda^*[R^*(b_1^2)]].$$

(A.6)

so that, from Theorem 1,

$$\lambda^*[R^*(b_1^2)] \geq \lambda^*(x).$$

(A.7)

Thus, from (A.4),

$$R^*(b_1^1) \leq x,$$

(A.8)

which, dividing (A.8) by $b_1^1$, implies

$$R^*(b_1^1)/b_1^1 \leq (1 + \bar{r})/[1 - \lambda^*[R^*(b_1^2)].$$

(A.9)

Combining (A.3) and (A.9)

$$R^*(b_i^2)/b_i^2 \geq R^*(b_i^1)/b_i^1.$$}

Thus $R^*(b_i)$ is convex. ||

APPENDIX B

A Note on the Maximum Likelihood Estimation of Models with Min Conditions

The notation of this appendix is entirely self-contained. The econometric problem of equation (42) of the text is to estimate $A_1$ and $A_2$ where

$$y_1 = A_1X_1 + u_1$$

$$y_2 = A_2X_2 + u_2$$

$$y = \min (y_1, y_2)$$

and where $u_1$ and $u_2$ are normally distributed random variables. We assume that $u_1$ and $u_2$ are also independently distributed with common variance $\sigma^2$.

The crucial aspect of the model (B.1) is that only $y$ is observed. It is not known a priori whether any $y$ is a $y_1$ or a $y_2$, yet it is possible to estimate the parameters of both equations,
i.e. the $A_1$ and $A_2$. Define

$$f_i(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{1}{2\sigma^2} (y - A_i X_i)^2 \right]$$  \hspace{1cm} (B.2)

$$F_i(y) = \int_y^\infty f_i(y_i) dy_i.$$  \hspace{1cm} (B.3)

It can be shown (Maddala and Nelson, 1974) that

$$H(y) = f_1(y)F_2(y) + f_2(y)F_1(y)$$  \hspace{1cm} (B.4)

is the likelihood of $y$. Maximizing this likelihood yields estimates of $A_1$, $A_2$ and $\sigma$.

Finally, the probability $\pi_i(y)$ that any given, observed $y$ belongs to regime $i$ is

$$\pi_i = f_i P_i / \sum_{j=1}^{2} f_j P_j,$$ \hspace{1cm} (B.5)

and is useful in classifying observed $y$'s into the two different regimes. This result is easily proved by manipulation of formulae for conditional probabilities, and has the intuitive rationale of the fraction of the likelihood of an observation which is contributed by the $i$-th regime. See Gersovitz (1980) for further discussion of classification in this model.

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**APPENDIX C**

**Definition of Variable Values**

**PRIDT**: Public debt (including undisbursed) with maturity over one year to suppliers, financial markets and other private creditors (World Bank, *World Debt Tables*) given in U.S. current dollars divided by the U.S. GNP deflator (Federal Reserve Board, *Bulletin*). The variable was then logged.

**$\sigma_x$**: For the periods 1964–1970 and 1968–1974 for each country, a regression of the natural logarithm of real exports (IMF, *International Financial Statistics*) on a constant and time was performed. $\sigma_x$ was defined as the standard error of this regression.

**M/Y**: Ratio of imports to GNP (IMF, *International Financial Statistics*).


**POP**: Total population (World Bank, *World Tables*). Logged.

**GY**: For the periods 1964–1970 and 1968–1974 for each country, a regression of the natural logarithm of real GNP (IMF, *International Financial Statistics*) on a constant and time was performed. $GY$ was defined as the coefficient of time.

**PUBDT**: Public debt (including undisbursed) to international organizations, DAC governments and other (non-communist) governments (World Bank, *World Debt Tables*) given in U.S. current dollars divided by the U.S. GNP deflator (Federal Reserve Board, *Bulletin*). The variable was then logged.
### APPENDIX D

Countries in the Sample and the Probability of the Supply Constrained ($\bar{b}$) Regime

<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
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First version received April 1978; final version accepted June 1980 (Eds.).

NOTES
2. Our model is related to and complements the work by Jaffee and Russell (1976) on bankruptcy by agents in a domestic economy, although our emphasis is quite different, being appropriate to the features of
international borrowing. Jaffee and Russell stress that lenders have difficulty distinguishing inherently honest from dishonest borrowers. They allow for a two-period structure with lending in the first period and then question whether loans will be repaid in the second period. Lenders can impose a credit ceiling to ensure that dishonest borrowers never borrow so much that the benefit of not repaying exceeds an exogenously specified cost of nonrepayment. Honest borrowers, since they cannot be so identified, are also credit rationed, being allowed less than they may wish.

3. The likelihood was maximized using the GRADX option of the GQOPT program at Princeton University based on the method of Goldfeld, Quandt and Trotter (1966). The coefficient estimates correspond to the highest value of the likelihood function which was reached.

4. In estimating the model of (42), we assumed that $u$ and $v$ have a common variance (Table I). This assumption avoided the problems associated with the global unboundedness of the likelihood of Appendix $B$ when both variances are estimated (Quandt, 1977).

An alternative approach to the global unboundedness problem is presented by Amemiya and Sen (1977). These authors prove that if a variance constraint is not imposed, a local maximum of the likelihood function provides consistent estimates of the parameters. Releasing the variance constraint yielded an estimate of $\sigma_u/\sigma_v = 1.37$ and a log likelihood of $-94.4$. This value yields a likelihood ratio test value of 1.8 compared with the $\chi^2$ value with 1 degree of freedom at 0.05 significance of 3.84 so that the hypothesis of equal error variances cannot be rejected.

5. We represented the effect of variability by the percent variability in exports, $\sigma_v$, rather than by the percent variability in income, $\sigma_u$. For most poor countries, exports are better measured than income. We also estimated the equations using $\sigma_u$ with basically similar results but with a lower log likelihood value ($-99.7$).

6. An alternative measure of trade dependence is the export-income ratio, $X/Y$. When we replaced $M/Y$ by $X/Y$, the value of the likelihood function declined (log likelihood $= -97.8$). The coefficient estimates were, however, similar.

7. The coefficients of $D$ could also reflect the inhomogeneity of the sample across the two years. Regressions on a homogeneous sample of the same 36 countries in the two years (72 observations) yielded coefficients on all variables differing negligibly from those presented in Tables I. The coefficients on the time dummy, in particular, were $-0.80$, and $1.27$ in the $b$, and $b^*$ equations, respectively. We conclude that sample inhomogeneity is not crucial to our results.

8. The log likelihood rose to $-85.6$ when all parameters were allowed to vary between the two years. This value yields a likelihood ratio test value of 19.2 compared with the $\chi^2$ value of 13 degrees of freedom at 0.05 significance of 22.4.

9. An ordinary least squares regression assuming all observations were known to be generated by only one regime yielded a log likelihood of $-103.9$ and an $R^2$ of 0.69. The hypothesis of one regime versus two regimes cannot be formulated as a nested hypothesis and the likelihood ratio test is not strictly applicable. Quandt (1978) has, however, suggested use of the likelihood ratio test despite its theoretical inappropriateness and presents sampling evidence indicating the practical usefulness of this test. In this case, the pseudo likelihood ratio test value is 17.4 which exceeds the value of $\chi^2$ with 8 degrees of freedom at 0.05 significance of 15.5, suggesting that a two regime model is superior.

For comparison, the OLS coefficient estimates in the order of Table I, with standard errors in brackets, are: $-3.11(1.59)$, $4.47(2.71)$, $1.77(1.68)$, $1.00(6.31)$, $-0.60(3.53)$, $-0.017(0.47)$, $0.606(3.07)$, and $-0.46(1.93)$.

REFERENCES


INTERNATIONAL MONETARY FUND International Financial Statistics.